General Instruction:

A. All **Questions** are compulsory.

B. This Question Paper contains **29 Questions**.

C. Questions **1-4** in **Section A** are **Very Short -Answer** type Questions carrying **1 Mark** each.

D. Questions **5-12** in **Section B** are **Short - Answer** type Questions carrying **2 Marks** each.

E. Questions **13-23** in **Section C** are **Long - Answer-I** type Questions carrying **4 Marks** each.

F. Question **24-29** in **Section D** are **Long - Answer-II** type Questions carrying **6 Marks** each.

Section A

1

Show that the relation R in the set {1, 2, 3} given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

[1]

2

For what values of k, the system of linear equations

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

has a unique solution?

[1]

3

find λ and μ if

$(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} + \lambda\hat{j} + \mu\hat{k}) = 0$

[1]

4

Show that the function f : N → N given by f(1) = f (2) = 1 and f (x) = x – 1, for every x > 2, is onto but not one-one.

[1]

Section B

5

Find $\frac{dy}{dx}$ at $x = 1, y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$

[2]

6

Evaluate the following definite integrals as limit of sums.

$\int_2^3 x^2 dx$

[2]

7

Show that all the diagonal elements of a skew symmetric matrix are zero.

[2]

8

Write the order and degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$

[2]

9

Give a condition that the three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ form the three sides of a triangle. What are the other possibilities?

[2]

10

Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six.

Find the probability that it is actually a six.

Do you also agree that the value of truthfulness leads to more respect in the society?

[2]

11

If $f(x) = \int_0^x t \sin t \, dt$, write the value of $f'(x)$

[2]

12

PQRS is a parallelogram. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices P, Q and R respectively with reference to the origin of reference O ; find the position vector of S with reference to O.

[2]

Section C

13

Find x such that the four points A(4, 1, 2), B(5, x, 6), C(5, 1, -1) and D(7, 4, 0) are coplanar.

[4]

OR

Find the coordinates of the foot of perpendicular drawn from the point A (-1,8,4) to the line joining the points B(0,-1,3) and C(2,-3,-1). Hence find the image of the point A in the line BC.

[4]

14

A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

[4]

15

If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{d^2y}{dx^2} = 0$

[4]

16

Show that the function $f(x) = |x - 3|, x \in \mathbb{R},$ is continuous but not differentiable at x=3.

[4]

OR

$$\text{If } f(x) = \begin{cases} \frac{\sin(a+1) + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx} - 1}{x}, & x > 0 \end{cases}$$

is continuous at x = 0, then find the values of a and b.

[4]

17

In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides

[4]

18

Find : $\int \frac{(2x-5)e^{2x}}{(2x-3)} dx$

[4]

19

Find the particular solution of the differential equation

$\frac{dy}{dx} = -\frac{x + y \cos x}{1 + \sin x}$ given that $y = 1$ when $x = 0$

[4]

OR

Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ is the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)\, dy$, where C is a parameter.

[4]

20

Prove that

[4]

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

21

Find all points of discontinuity of f where

[4]

$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if } x \geq 0 \end{cases}$$

22

Show that the four points A, B, C and D with position vectors

$4\hat{i} + 5\hat{j} + \hat{k}, -\hat{j} - \hat{k}, 3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.

[4]

23

Find the value(s) of x for which y = $[x(x-2)]^2$ is an increasing function.

[4]

Section D

24

Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

[6]

25

Let A= R × R and * be a binary operation on A defined by

(a, b) * (c, d) = (a+c, b+d)

Show that * is commutative and associative. Find the identity element for *

on A. Also find the inverse of every element (a, b) ∈ A.

[6]

26

By using the properties of definite integrals, evaluate the following:

$$\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$$

[6]

27

Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area using integration.

[6]

OR

Using the method of integration, find the area of the triangular region whose vertices are (2, -2), (4, 3) and (1, 2).

[6]

28

Minimum and maximum z = 5x + 2y subject to the following constraints:

x – 2y ≤ 2

3x + 2y ≤ 12

−3x + 2y ≤ 3

x ≥ 0, y ≥ 0

[6]

A retired person wants to invest an amount of Rs. 50, 000. His broker recommends investing in two type of bonds ‘A’ and ‘B’ yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs. 20,000 in bond ‘A’ and at least Rs. 10,000 in bond ‘B’. He also wants to invest at least as much in bond ‘A’ as in bond ‘B’. Solve this linear programming problem graphically to maximise his returns.

[6]

29

Show that lines:

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 4\hat{j} + 2\hat{k} + \mu(2\hat{j} - \hat{j} + 3\hat{k})$ are coplanar.

Also, find the equation of the plane containing these lines.

[6]