**General Instruction:**

A. All **Questions** are compulsory.

B. This Question Paper contains **29 Questions**.

C. Questions **1-4** in **Section A** are **Very Short -Answer** type Questions carrying **1 Mark** each.

D. Questions **5-12** in **Section B** are **Short - Answer** type Questions carrying **2 Marks** each.

E. Questions **13-23** in **Section C** are **Long - Answer-I** type Questions carrying **4 Marks** each.

F. Questions **24-29** in **Section D** are **Long - Answer-II** type Questions carrying **6 Marks** each.

## Section A

1

Let A be the set of all students of a boys school. Show that the relation R in A given by R = {(a, b) : a is sister of b} is the empty relation and R' = {(a, b) : the difference between heights of a and b is less than 3 meters} is the universal relation.

[1]

2

Evaluates :

[1] $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

3

If $P_1$ , $P_2$, $P_3$, $P_4$ are points in a plane or space and O, the origin of vectors, show that $P_4$ coincides with O if an only if

[1] $\overrightarrow{OP_1} + \overrightarrow{P_1P_2} + \overrightarrow{P_2P_3} + \overrightarrow{P_3P_4} = \vec{0}$.

4

Show that the function f : N → N given by f(x) = 2x, is one-one but not onto.

[1]

## Section B

5

Using principle value, evaluate the following :

[2] $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

6

Construct a 3 x 4 matrix whose elements are

[2]



7

Examine whether the function f given by $f(x) = x^2$ is continuous at x = 0.

[2]

8

The cost function C(x), in rupees, of producing x items ($x \geq 15$) in a certain factory is given by $C(x) = 20 + 10x^2 + \frac{15}{x}$. Determine the marginal cost function and the marginal cost of producing 100 items.

[2]

9

Evaluate the following integral:

[2] $\int_0^4 \frac{dx}{\sqrt{x^2+2x+3}}$

10

Determine the order and degree of the differential equation:

[2] $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 5\frac{d^2y}{dx^2}$

11

Find values of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

[2]

12

Find the angle between the vector $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.

[2]

## Section C

13

[4] Evaluate the determinant

$\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin & 1 & \sin\theta \\ -1 & -\sin & 1 \end{vmatrix}$, Also, prove that $2 < A < 4$.

OR

Evaluate $\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

[4]

14

Discuss continuity of the function f given by

f(x) = | x – 1| + | x – 2 ] at x = 1 and x = 2.

[4]

15

The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters?

[4]

16

Show that the following differential equation is homogeneous and find a primitive of it. Derive the solution wherever possible:

$y' - y + x \sin\left(\frac{y}{x}\right) = 0$

[4]

OR

Solve $\left(x \sin\frac{y}{x}\right)dy = \left(y \sin\frac{y}{x} - x\right)dx$

[4]

17

Find all the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} x+2, & \text{if } x < 1 \\ 0, & \text{if } x = 1 \\ x-2, & \text{if } x > 1 \end{cases}$$

[4]

18

Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx.$

[4]

19

If P, Q, R, S are the points (– 2, 3, 4), (– 4, 4, 6), (4, 3, 5), (0, 1, 2), prove by projection that PQ is perpendicular to RS.

[4]

OR

Show that the line through the points (1, – 1, 2), (3, 4, – 2) is perpendicular to the line through the

points (0, 3, 2) and (3, 5, 6).

[4]



20

A die is thrown three times. Events A and B are defined as below:

A: 4 on the third throw

B: 6 on the first and 5 on the second throw

Find the probability of A given that B has already occurred.

[4]

21

A point source of light along a straight road is at a height of 'a' metres. A boy 'b' metres in height is walking along the road. How fast is his shadow increasing if he is walking away from the light at the rate of c metres per minute?

[4]

22

ABCDEF is a regular hexagon. Show that

(i) $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} = 3\vec{AD}$

(ii) $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} = 6\vec{AO}$

where O is centre of the hexagon.

[4]

23

An unbiased die is thrown twice. Let the event A be 'odd number on the first throw' and B the event 'odd number on the second throw'. Check the independence of the events A and B.

[4]

## Section D

24

Show that each of the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12 }, given by

(i) R = {(a, b) : | a – b | is a multiple of 4 }

(ii) R = {(a, b) : a = b} is an equivalence relation. Find the set of all elements related to 1 in each case.

[6]

25

[6] Prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

27

Evaluate:

[6] $\int_0^{\frac{\pi}{2}} \frac{dx}{a\cos x + b\sin x}, a, b > 0$

27

Find the area of the region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4} = 1.$

[6]

OR

Draw a graph of $\frac{x^2}{9}+\frac{y^2}{25} = 1$ and evaluate area bounded by it.

[6]

28

A (– 1, 2, – 3), B (5, 0, – 6), C (0, 4, – 1) are three points. Show that the direction cosines of the bisectors of are proportional to 25, 8, 5 and -11, $AB = \sqrt{(5+1)^2+(0-2)^2+(-6+3)^2}$
$= \sqrt{36+4+9} = \sqrt{49} = 7$
$AC = \sqrt{(0+1)^2+(4-2)^2+(-1+3)^2}$
$= \sqrt{1+4+4} = \sqrt{9} = 3$

[6]

OR

For the cartesian and vector equation of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}.$

[6]

29

Solve the following linear programming problem graphically:

Maximise Z = 4x + y

subject to the constraints: x + y ≤ 50, 3x + y ≤ 90, x ≥ 0, y ≥ 0

[6]